

# A more efficient way of finding Hamiltonian cycle

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## 1 Introduction

Algorithm tests if a Hamiltonian cycle exists in directed graphs, if it exists algorithm can show found Hamiltonian cycle. If you want to test an undirected graph, such a graph should be converted to the form of directed graph. Previously known algorithm solving Hamiltonian cycle problem - brute-force search can't handle relatively small graphs. Algorithm presented here is referred to simply as "algorithm" in this paper.

### **Why algorithm is more efficient than brute-force search?**

In order to find Hamiltonian cycle, algorithm should find edges that creates a Hamiltonian cycle. Higher number of edges creates more possibilities to check to solve the problem. Both brute-force search and algorithm use recursive depth-first search. The reason why brute-force search often fails to solve this problem is too large number of possibilities to check in order to solve the problem.

Algorithm rests on analysis of original graph and opposite graph to it. Algorithm prefers "to think over" which paths should be checked than check many wrong paths. Algorithm is more efficient than brute-force search because it can:

- **remove unnecessary edges from graph(3)**
- **test when Hamiltonian cycle can't exist in graph(4)**
- **choose most optimal path(7)**

## 2 Definitions

1. **Graph** - set of vertices  $v_1, v_2, \dots, v_N$  and edges  $(a_1, a_2), (a_3, a_4), \dots, (a_{M-1}, a_M)$  where  $\{a_1, a_2, \dots, a_M\} \in \{v_1, v_2, \dots, v_N\}$
2. **Path** - set of edges  $(v_1, v_2), (v_2, v_3), \dots, (v_{K-1}, v_K)$  where  $v_1 \neq v_2 \neq \dots \neq v_K$  and graph contains edges  $(v_1, v_2), (v_2, v_3), \dots, (v_{K-1}, v_K)$
3. **Cycle** - Path from graph which contains an edge  $(v_K, v_1)$  where  $v_K$  is last vertex in cycle and  $v_1$  is first vertex in cycle.
4. **Hamiltonian cycle** - cycle of length equal to the number of vertices in graph
5. **Hamiltonian graph** - graph which contains Hamiltonian cycle
6. **Vertex degree** - Vertex  $v_1$  has degree equal to  $W$  if graph contains following  $W$  edges  $(v_1, b_1), (v_1, b_2), \dots, (v_1, b_W)$
7. **Opposite graph** - Graph  $G$  is set of vertices  $v_1, v_2, \dots, v_N$  and edges  $(a_1, a_2), (a_3, a_4), \dots, (a_{M-1}, a_M)$ . Graph opposite to  $G$  is set of vertices  $v_1, v_2, \dots, v_N$  and edges  $(a_2, a_1), (a_4, a_3), \dots, (a_M, a_{M-1})$

Example:

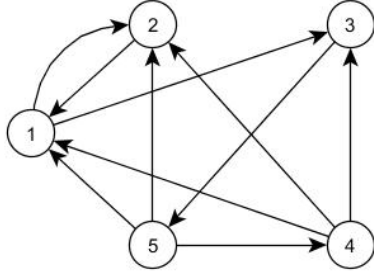


Figure 1: Original graph

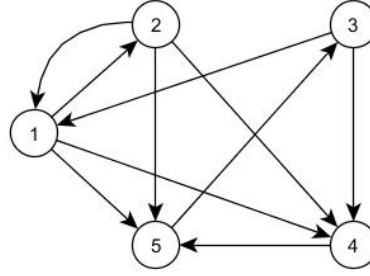


Figure 2: Opposite graph

## 3 Rules of removing unnecessary edges from graph

### 3.1 No loops and multiple edges

Algorithm removes multiple edges and loops from graph because they are irrelevant in process of finding Hamiltonian cycle.

### 3.2 Unique neighbours

Problem of finding Hamiltonian cycle in graph with  $N$  vertices can be described as a problem of finding following edges:

$$a_1 \rightarrow x_1$$

$$a_2 \rightarrow x_2$$

$$\vdots$$

$$a_N \rightarrow x_N$$

where  $x_1, x_2, \dots, x_N \in \{a_1, a_2, \dots, a_N\}$ ;  $x_1 \neq x_2 \neq \dots \neq x_N$  and edges  $a_1 \rightarrow x_1, a_2 \rightarrow x_2, \dots, a_N \rightarrow x_N$  don't create a cycle that is not Hamiltonian.

For graph with  $N$  vertices algorithm should find  $N$  unique neighbours.

Let's consider example graph presented with adjacency list:

1 : 3, 4

2 : 3, 4

3 : 2, 4

4 : 1, 3

Let's test which neighbours have vertices 1 and 2. They are:

3, 4, 3 and 4.

Let's test how many and which unique neighbours are on the list.

There are 2 vertices: 3 and 4.

**When for  $M$  tested vertices the number of unique neighbours equals  $M$  then algorithm can remove these unique neighbours from vertices, that were not tested.  $M$  is lower than  $N$ .**

Graph after removal of unnecessary edges presented with adjacency list:

1 : 3, 4

2 : 3, 4

3 : 2

4 : 1

"Unique neighbours" test is used for original graph and opposite graph.

#### 3.2.1 Method of accomplishing "unique neighbours" test

For graph  $G$ , for every vertex  $V$  in  $G$  algorithm takes its adjacency list and checks how many and which of others adjacency lists are subsets of  $V$ 's adjacency list.

Let's consider example graph presented with adjacency list:

```
1 : 4, 8      5 : 1, 4, 8
2 : 1, 3, 5, 7  6 : 1, 3, 4, 5
3 : 4, 5, 6    7 : 3, 5
4 : 1, 3, 7    8 : 5, 7
```

To analyze vertex 2 with "unique neighbours" test algorithm takes its adjacency list - 1, 3, 5, 7 and checks how many and which of others adjacency lists are subsets of 2's adjacency list, they are:

```
4 : 1, 3, 7    7 : 3, 5    8 : 5, 7
```

Algorithm can remove edges marked with red color:

```
1 : 4, 8      5 : 1, 4, 8
2 : 1, 3, 5, 7  6 : 1, 3, 4, 5
3 : 4, 5, 6    7 : 3, 5
4 : 1, 3, 7    8 : 5, 7
```

### 3.2.2 Add edges to remove edges

Let's consider example graph presented with adjacency list:

```
1 : 5, 6, 7    5 : 2, 7
2 : 1, 4        6 : 3, 2, 5
3 : 2, 5, 7    7 : 1, 4, 5
4 : 2, 5, 6
```

When algorithm tests graph with method 3.2.1 it will not find any edges that it could remove. However with addition of edge  $1 \rightarrow 2$  algorithm can remove edges marked with red color:

```
1 : 2, 5, 6, 7  5 : 2, 7
2 : 1, 4        6 : 3, 2, 5
3 : 2, 5, 7    7 : 1, 4, 5
4 : 2, 5, 6
```

Temporary addition of new edges may allow algorithm to remove some edges.

### 3.2.3 Which edges should be added?

Let's consider graph with  $N$  vertices:  $x_1, x_2, \dots, x_N$ . To test which edges should be added while testing vertex  $x_1$  with method 3.2.1 algorithm compares adjacency list of  $x_1$  with adjacency lists of  $x_2, x_3, \dots, x_N$ :

If lists  $x_1$  and  $x_2$  have at least one common vertex then added edges are these that are on the list  $x_2$  but that are not on list  $x_1$ .

If lists  $x_1$  and  $x_3$  have ...

⋮

If lists  $x_1$  and  $x_N$  have ...

Let's consider example graph presented with adjacency list:

1 : 2, 5, 6, 7    5 : 2, 4, 7, 8  
 2 : 3, 5, 7    6 : 3, 4, 5, 7, 8  
 3 : 2, 5, 7    7 : 2, 5, 8  
 4 : 1, 2, 7, 8    8 : 1, 3, 5

Added edges(edges marked with red color are not tested):

-	1	2	3	4	5	6	7	8
1	-	1 → 3		1 → 8	1 → 4 1 → 8		1 → 8	1 → 3
2		-					2 → 8	2 → 1
3			-				3 → 8	3 → 1
4				-			4 → 5	4 → 3 4 → 5
5					-			
6		6 → 1	6 → 2 6 → 1		6 → 2	-	6 → 2	6 → 1
7							-	7 → 1 7 → 3
8								-

### 3.2.4 Combination of added edges

Let's consider example graph presented with adjacency list:

1 : 9, 12    6 : 4, 5, 7, 9, 10, 12    11 : 3, 4, 6, 10  
 2 : 1, 5, 12, 13    7 : 2, 6, 11, 12, 14    12 : 3, 5, 6, 7  
 3 : 4, 12    8 : 4, 7, 10    13 : 3, 5, 8, 11, 14  
 4 : 6, 9, 10, 12    9 : 4, 6, 13, 14    14 : 3, 5, 7  
 5 : 1, 4, 7, 8    10 : 3, 4, 8, 11, 14

Algorithm can temporary add a combination of added edges.

Such a method allows algorithm to create following set of edges: 1 → 3, 1 → 4, 1 → 5, 1 → 6, 1 → 7, 1 → 10 which allows algorithm to remove edges marked with red color:

1 : 3, 4, 5, 6, 7, 9, 10, 12    6 : 4, 5, 7, 9, 10, 12    11 : 3, 4, 6, 10  
 2 : 1, 5, 12, 13    7 : 2, 6, 11, 12, 14    12 : 3, 5, 6, 7  
 3 : 4, 12    8 : 4, 7, 10    13 : 3, 5, 8, 11, 14  
 4 : 6, 9, 10, 12    9 : 4, 6, 13, 14    14 : 3, 5, 7  
 5 : 1, 4, 7, 8    10 : 3, 4, 8, 11, 14

### 3.3 Single edge in only one direction

When in original graph or in opposite graph the only one neighbour of vertex  $X$  is vertex  $Y$ , it means that for Hamiltonian cycle to exist edge  $X \rightarrow Y$  must be in this cycle, when graph also contains edge  $Y \rightarrow X$  then algorithm can remove edge  $Y \rightarrow X$ .

Example graph presented with adjacency list:

1 : 2, 3, 4

2: 3

3: 2, 4

4 : 1

Algorithm can remove edge  $3 \rightarrow 2$ .

## 4 When algorithm stops or don't start the search for Hamiltonian cycle

### 4.1 Cycle that is not Hamiltonian

#### 4.1.1 Vertices with 1 neighbour

In original graph or in opposite graph among vertices that have 1 neighbour exists cycle, that is not Hamiltonian. Example graph:

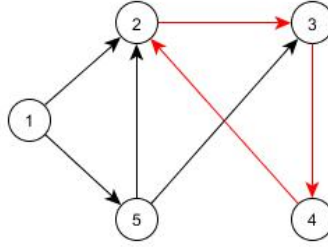


Figure 3: Cycle created by vertices: 2, 3 and 4

#### 4.1.2 Vertices with more than 1 neighbour

Let's consider graph with  $N$  vertices. Graph contains vertex  $X$  that has  $M$  neighbours,  $1 < M < N/2$ ,  $X$  has the same list of in-neighbours and out-neighbours. Graph also contains  $M - 1$  vertices with the same list of in-neighbours and out-neighbours and list of neighbours which is a subset of list of neighbours of vertex  $X$ .

Example 1. presented with adjacency list:

Original and opposite graph:

X: A B

Y: A B

There are only 2 paths that can be created, they are:

- $A \rightarrow X \rightarrow B \rightarrow Y \rightarrow A$
- $B \rightarrow X \rightarrow A \rightarrow Y \rightarrow B$

Every possible path creates cycle that is not Hamiltonian.

Example 2. presented with adjacency list:

Original and opposite graph:

X: A B C

Y: A B C

Z: A B C

There are 12 paths that can be created, they are:

- $A \rightarrow X \rightarrow B \rightarrow Y \rightarrow C \rightarrow Z \rightarrow A$
- $A \rightarrow X \rightarrow B \rightarrow Z \rightarrow C \rightarrow Y \rightarrow A$
- $A \rightarrow X \rightarrow C \rightarrow Y \rightarrow B \rightarrow Z \rightarrow A$
- $A \rightarrow X \rightarrow C \rightarrow Z \rightarrow B \rightarrow Y \rightarrow A$
- $B \rightarrow X \rightarrow A \rightarrow Y \rightarrow C \rightarrow Z \rightarrow B$
- $B \rightarrow X \rightarrow A \rightarrow Z \rightarrow C \rightarrow Y \rightarrow B$
- $B \rightarrow X \rightarrow C \rightarrow Y \rightarrow A \rightarrow Z \rightarrow B$
- $B \rightarrow X \rightarrow C \rightarrow Z \rightarrow A \rightarrow Y \rightarrow B$
- $C \rightarrow X \rightarrow A \rightarrow Y \rightarrow B \rightarrow Z \rightarrow C$
- $C \rightarrow X \rightarrow A \rightarrow Z \rightarrow B \rightarrow Y \rightarrow C$
- $C \rightarrow X \rightarrow B \rightarrow Y \rightarrow A \rightarrow Z \rightarrow C$
- $C \rightarrow X \rightarrow B \rightarrow Z \rightarrow A \rightarrow Y \rightarrow C$

Every possible path creates cycle that is not Hamiltonian.

Example 3. presented with adjacency list:

Original and opposite graph:

X: A B C

Y: A B

Z: A C

There are 2 paths that can be created, they are:

- $B \rightarrow X \rightarrow C \rightarrow Z \rightarrow A \rightarrow Y \rightarrow B$
- $C \rightarrow X \rightarrow B \rightarrow Y \rightarrow A \rightarrow Z \rightarrow C$

Every possible path creates cycle that is not Hamiltonian.

## 4.2 Not enough unique neighbours

Number of unique neighbours checked for  $M$  vertices is lower than  $M$ .

Example presented with adjacency list:

1 : 4, 5

2 : 4, 5

3 : 4, 5

4 : 1, 2

5 : 1, 3

## 4.3 Graph is disconnected

Let's consider graph with  $N$  vertices:  $a_1, a_2, \dots, a_N$ . A path doesn't exist between vertices:  $a_1$  to  $a_2$  or  $a_1$  to  $a_3 \dots$  or  $a_1$  to  $a_N$ .

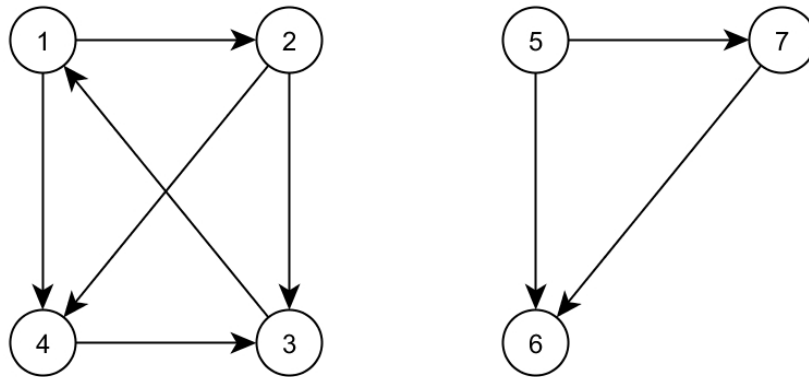


Figure 4: A path doesn't exist between vertices 1 and 5



## 5 Edges constantly removed

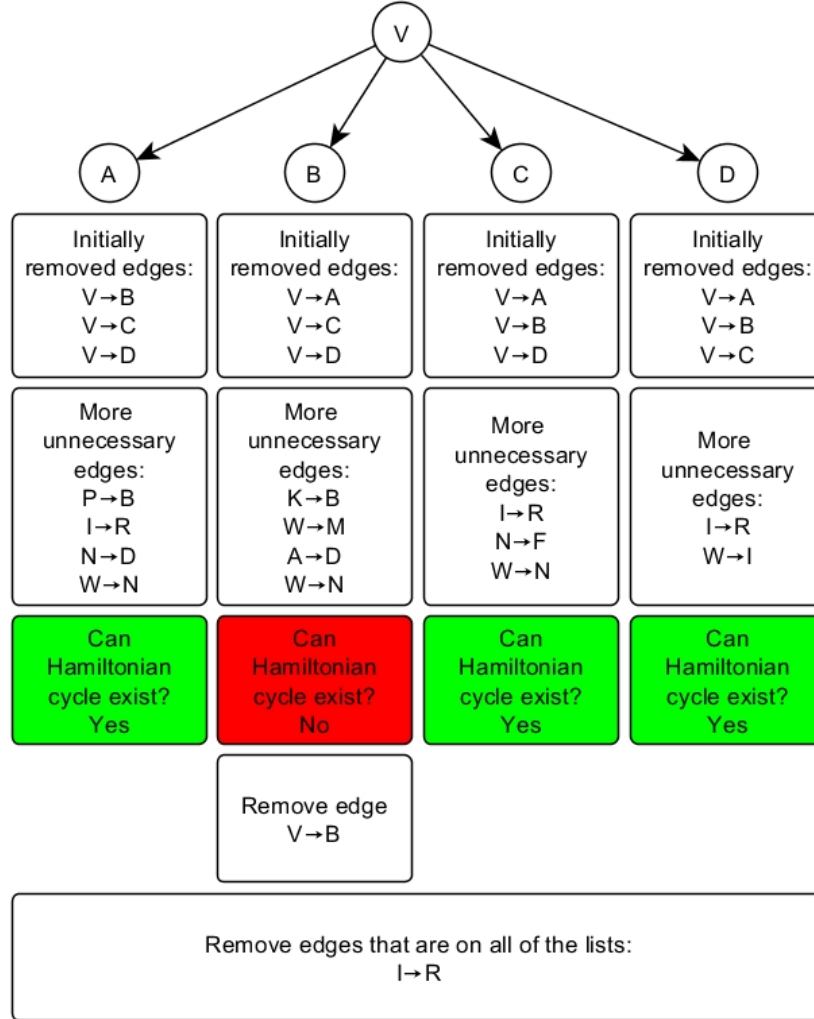


Figure 5:  $I \rightarrow R$  is an edge constantly removed

Graph with  $N$  vertices:  $a_1, a_2, \dots, a_N$ . For Hamiltonian cycle to exist each of these vertices must contain an edge that is in this cycle. Vertex  $V$  has  $M$  neighbours:  $b_1, b_2, \dots, b_M$ ;  $M \geq 2$ ;  $b_1, b_2, \dots, b_M \in \{a_1, a_2, \dots, a_N\}$ . After removal of certain edges, algorithm may be able to remove more unnecessary edges. Following tests are made for every vertex that has more than 1 neighbour, tests are based on an assumption that chosen edge must be in Hamiltonian cycle:

- Chosen edge:  $V \rightarrow b_1$ . Test which edges will be removed after removal of edges:  $V \rightarrow b_2, V \rightarrow b_3, \dots, V \rightarrow b_M$ , all removed edges are saved on list  $l_1$
- Chosen edge:  $V \rightarrow b_2$ . Test which edges will be removed after removal of edges:  $V \rightarrow b_1, V \rightarrow b_3, \dots, V \rightarrow b_M$ , all removed edges are saved on list  $l_2$
- ...
- Chosen edge:  $V \rightarrow b_M$ . Test which edges will be removed after removal of edges:  $V \rightarrow b_1, V \rightarrow b_2, \dots, V \rightarrow b_{M-1}$ , all removed edges are saved on list  $l_M$

If test of following assumption: edge  $V \rightarrow b_x$ , where  $x \in \{1, 2, \dots, M\}$ , must be in Hamiltonian cycle, proves that Hamiltonian cycle can't exist if edge  $V \rightarrow a_x$  is in it, then algorithm can remove edge  $V \rightarrow b_x$  and doesn't save removed edges on list  $l_x$ . Example: edge  $V \rightarrow B$  shown in Figure 5.

When for all of the edges:  $V \rightarrow b_1, V \rightarrow b_2, \dots, V \rightarrow b_M$  test will prove that Hamiltonian cycle can't exist if chosen edge is in it, then graph does not contain Hamiltonian cycle.

Edges that are on all of the lists:  $l_1, l_2, l_3, \dots, l_M$  can be removed from graph because for Hamiltonian cycle to exist at least one of the tested edges must be chosen and regardless which edge will it be, the edges that are on all of the lists will be constantly removed. Example: edge  $I \rightarrow R$  shown in Figure 5.

## 6 Reversed graph

Let's consider graph  $G$  with  $N$  vertices. Reversed graph of  $G$  is created by changing every edge  $a \rightarrow b$  in  $G$  to  $(N - a) \rightarrow (N - b)$

Example:

1:	-	-	3	4	-	-	-	-	-	--
2:	-	-	3	-	-	-	-	-	9	--
3:	1	2	-	-	-	-	-	-	-	--
4:	1	-	-	-	5	-	-	-	-	--
5:	-	-	-	4	-	-	-	8	-	--
6:	-	-	-	-	-	-	7	-	-	10
7:	-	-	-	-	-	6	-	-	9	--
8:	-	-	-	-	5	-	-	-	-	10
9:	-	2	-	-	-	-	7	-	-	--
10:	-	-	-	-	-	6	-	8	-	--

Figure 6: Original graph

1:	-	-	3	-	5	-	-	-	-	--
2:	-	-	-	4	-	-	-	-	9	--
3:	1	-	-	-	-	6	-	-	-	--
4:	-	2	-	-	5	-	-	-	-	--
5:	1	-	-	4	-	-	-	-	-	--
6:	-	-	3	-	-	-	7	-	-	--
7:	-	-	-	-	-	6	-	-	-	10
8:	-	-	-	-	-	-	-	-	9	10
9:	-	2	-	-	-	-	-	8	-	--
10:	-	-	-	-	-	-	7	8	-	--

Figure 7: Reversed graph

If reversed graph of  $G$  is Hamiltonian then  $G$  is Hamiltonian.

## 7 Most optimal path

### 7.1 Visiting a path

Both brute-force search and algorithm use recursive depth-first search to test paths - test possibilities, however the second one use it differently. Algorithm's goal is to find edges described in the beginning of 3.2.

Let's consider graph  $G$  with  $N$  vertices and path  $P$  from graph  $G$  which consists of following edges:  $a_1 \rightarrow a_2, a_2 \rightarrow a_3, \dots, a_{M-1} \rightarrow a_M$ .

**Brute-force search** will visit  $P$  in following way:

1. Visit vertex  $a_1$
2. Visit vertex  $a_2$
- $\vdots$
- M. Visit vertex  $a_M$
- M+1. Check if  $N == M$  and if  $G$  contains edge  $a_M \rightarrow a_1$

**Algorithm** will visit  $P$  in following way:

1. Visit edge  $a_{x_1} \rightarrow a_{x_2}$
2. Remove unnecessary edges and test if Hamiltonian cycle can exist in graph
3. Test if Hamiltonian cycle was found
- If Hamiltonian cycle was not found:
4. Visit edge  $a_{x_3} \rightarrow a_{x_4}$
5. Remove unnecessary edges ...
- $\vdots$

$$x_1, x_2, \dots, x_N \in \{1, 2, \dots, M\}$$

Algorithm doesn't visit a path by visiting one vertex after another.

Algorithm doesn't necessarily need to visit every edge in path to know if it is not a Hamiltonian cycle or if it is a Hamiltonian cycle.

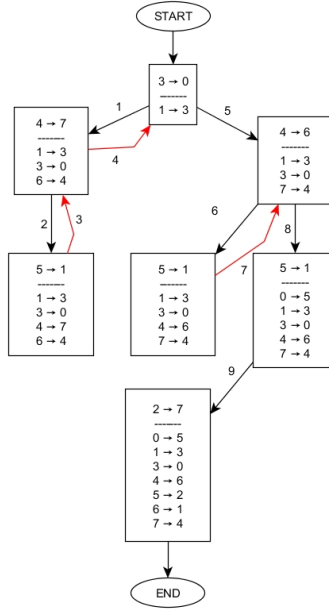


Figure 8: Example of usage of algorithm

## 7.2 Correct order

Let's consider graph  $G$ .  $G$  contains vertices: 2 and 3, vertex 2 has 5 neighbours and vertex 3 has 3 neighbours. Graph  $G$  has only one Hamiltonian cycle. Algorithm will test first vertex 3 because it will give algorithm probability equal to  $1/3$  of choosing edge that is in Hamiltonian cycle, which is better than probability equal to  $1/5$ .

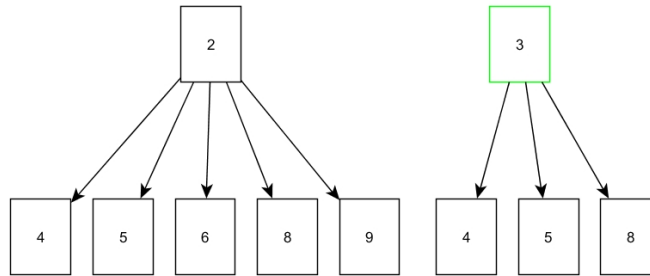


Figure 9: Algorithm will test first vertex 3

**When algorithm decide which vertex should it test first, it will decide to choose vertex with smaller degree.**

Algorithm creates **correct order**, it is a list of vertices in graph ordered by their degrees in ascending order.

### 7.3 Start vertex

Algorithm starts the search in graph with:

1. first vertex in correct order from graph with degree greater than 1, if such a vertex doesn't exist it will start with first vertex in graph
2. first vertices in correct order

### 7.4 Next edge

When algorithm tests vertex  $A$ , it have to decide which of  $A$ 's edges should it test first.

Let's consider following situation: vertex  $A$  has 3 neighbours:  $N, M$  and  $P$ , their degrees in opposite graph are:  $N - 5, M - 4$  and  $P - 8$ . Algorithm will test first edge  $A \rightarrow M$  because it will give algorithm probability equal to  $1/4$  of choosing edge that is in Hamiltonian cycle. To check edges in such an order, adjacency list of currently visited vertex is sorted by correct order from opposite graph.

### 7.5 Next vertex

Next visited vertex is selected:

1. as in 7.3.1
2. second vertex in currently tested edge

## 8 Features

1. Attempt of removal of unnecessary edges and testing if Hamiltonian cycle can exist in graph occurs:
  - (a) before search for Hamiltonian cycle begins.
  - (b) with every recursive call of function that searches for Hamiltonian cycle - with visiting every edge in tested path.
2. When algorithm makes the decision to test edge  $A \rightarrow X$  it removes all of the other edges from  $A$ .
3. Edges removed with choosing the wrong edge are restored.
4. Edges removed with choosing the wrong path are restored.

## 9 Algorithm

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**Algorithm 1** Algorithm

---

```
START
for stage = 1 to 5 do
  Initialize variables
  Remove multiple edges and loops
  Analyze graph
  if Hamiltonian cycle can't exist then
    END, Answer = "NO"
  end if
  FindHamiltonianCycle(7.3)
  if Answer was found then
    END
  end if
end for
```

---

### 9.1 Stages

Analysis of graph is accomplished with rules described in 3, 4 and 7 with following exceptions regarding stages:

1. 7.3 and 7.5
2. 3.2.3
3. 3.2.4 - only in 8.1a
4. 5 - only in 8.1a

Stage	1	2	3	4
1	1	no	no	no
2	2	no	no	no
3	1	yes	yes	no
4	1	yes	no	yes
5	1	no	no	yes

In 5. stage algorithm tests 6.

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**Algorithm 2** FindHamiltonianCycle(Vertex A)

---

```
level  $\leftarrow$  level + 1
if Hamiltonian cycle was found then
    Answer = "Hamiltonian cycle was found"
end if
Sort neighbours of A //7.4
for  $i = 0; i < A's \text{ degree}; i \leftarrow i + 1$  do
    B  $\leftarrow$  A's  $i$ -th neighbour
    C  $\leftarrow$  current graph
    Remove edge A  $\rightarrow$  B
    D  $\leftarrow$  edges: A  $\rightarrow$  all neighbours of A in current graph
    Remove edges D //8.2
    Add edge A  $\rightarrow$  B
    AnalyzeGraph(D) //8.1b
    if Hamiltonian cycle can exist then
        FindHamiltonianCycle(7.5)
        Restore graph to C //8.4
    else
        Add edges D //8.3
    end if
end for
level  $\leftarrow$  level - 1
if level == 0 then
    Answer = "Hamiltonian cycle doesn't exist in graph"
end if
```

---

## 9.2 Which vertices should be analyzed?

Before search for Hamiltonian cycle begins every vertex should be analyzed. In every recursive call of function that searches for Hamiltonian cycle two types of vertices should be analyzed:

1. vertices whose adjacency list where changed in current call of function
2. vertices whose adjacency list are supersets of any of adjacency list of 1

## 10 Examination of algorithm

Algorithm was tested on 4 types of graphs:

- "directed regular"
- "directed irregular"
- "undirected regular"
- "undirected irregular"

Graph "undirected" is a graph which has many edges  $X \rightarrow Y$  and also  $Y \rightarrow X$ . Graph "irregular" is a graph with median of all vertices degrees being much different than degree of vertex with highest number of neighbours.

```

0: 0 - - - - 6 7 8 - 10 11 12 13 -- -- 16 -- 18 -- -- -- 22 -- --
1: - - - - - 8 - -- -- -- 14 -- -- -- 18 19 -- 21 -- -- 24
2: 0 1 2 3 - 5 - 7 - - 10 -- 12 13 14 -- -- -- 19 -- -- -- 24
3: - 1 - - 4 - 6 - 8 - -- 11 -- -- -- -- 17 18 -- -- 21 -- --
4: - - 2 - - 5 - 7 8 - -- -- -- 13 14 15 -- 17 -- -- 20 -- 22 --
5: - 1 - - 4 - - - - -- -- -- 14 15 -- -- 18 -- -- -- 23 --
6: 0 1 - - 4 - 6 - 8 9 -- 11 12 -- -- -- -- 18 -- 20 -- -- 23 --
7: 0 - - 3 - - - - 9 -- -- -- -- -- -- -- 21 22 -- --
8: 0 1 - - - - 7 - 9 -- 11 -- 13 14 15 -- -- -- 19 -- -- 22 --
9: 0 - 2 - 4 5 - - - - -- 11 -- -- -- -- 16 17 -- -- 20 -- --
10: - - - 3 - 5 - - - - -- 12 -- -- -- 15 16 17 18 -- -- -- 24
11: 0 1 - - - - 7 - - -- 11 -- -- -- -- -- -- -- 24
12: - - 2 - - - 6 - - - -- 11 12 13 14 -- -- -- 18 19 -- 21 22 -- 24
13: - - - 3 - - - 7 8 - - - -- 14 -- -- -- 17 18 -- 20 -- --
14: - 1 2 - 4 - - - 8 9 -- 11 12 13 -- -- -- -- 19 -- 21 -- -- 24
15: - - - 4 - - - - -- 11 12 -- -- 15 -- -- -- 19 -- 21 -- -- 24
16: - 1 - - - - - 8 9 -- -- 12 13 -- -- -- -- -- -- -- 23 24
17: - 1 - 3 - - - - 8 - -- -- -- -- -- 17 18 19 -- -- 22 -- 24
18: - - - 3 4 - - - 8 9 -- 11 -- -- -- -- -- 17 -- 19 20 -- 22 --
19: 0 1 - - - - - 8 - -- -- 12 -- -- -- -- 18 -- -- -- 23 --
20: - - - - 5 - 7 - - -- 11 12 13 -- 15 16 -- 18 19 -- -- 24
21: - - - 3 - 6 - - - 10 -- -- -- -- -- 17 -- -- -- 22 -- 24
22: - - - 3 - 5 - - 8 9 10 -- -- 13 -- -- -- -- -- 20 21 -- 23 --
23: - - - 3 4 - 6 - 8 9 -- -- -- -- -- -- -- 22 -- --
24: - - - - - 7 8 - -- -- 12 13 -- -- -- -- -- 20 -- 22 --

```

Figure 10: Example of "directed regular" graph



```

0: - - 2 3 - - 6 - 8 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- 24
1: - - 2 3 - - 6 - 8 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
2: - - 2 3 - 5 - - - 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
3: - - 2 3 - - - - - 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
4: 0 - 2 3 - - - - - 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
5: - - 2 3 4 - - - - 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
6: - - 2 3 - - - - - 9 10 -- -- 13 14 15 16 -- -- 19 -- 21 -- -- --
7: - - 2 3 - - - - 8 9 10 -- 12 -- 14 15 16 -- -- 19 -- 21 -- -- --
8: - - 2 3 - - 6 - 8 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
9: - - 2 3 - - - - - 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
10: - - 2 3 - - 6 - 8 9 10 -- -- -- 14 15 16 -- -- 19 20 21 -- -- --
11: - - 2 3 - - - - 8 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 22 -- --
12: - - 2 3 - - 6 - 8 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
13: - - 2 3 - - - - 8 9 10 -- -- -- 14 15 16 17 -- 19 -- 21 -- -- --
14: - - 2 3 - - 6 - 8 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
15: - - 2 3 - - - - 8 9 10 -- -- -- 14 15 16 -- 18 19 -- 21 -- -- --
16: - - 2 3 - - 6 - 8 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
17: - - 2 3 - - - - 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
18: - 1 2 3 - - - - 8 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
19: - - 2 3 - - 6 - 8 9 10 11 -- -- 14 15 16 -- -- 19 -- 21 -- -- --
20: - - 2 3 - - - - 8 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- 23 --
21: - - 2 3 - - 6 - 8 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
22: - - 2 3 - - 6 7 8 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
23: - - 2 3 - - 6 - 8 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --
24: - - 2 3 - - 6 - 8 9 10 -- -- -- 14 15 16 -- -- 19 -- 21 -- -- --

```

Figure 11: Example of "directed irregular" graph

```

0: - - 2 - 4 - 6 7 - 9 10 -- 12 -- 14 15 16 -- -- -- 22 -- --
1: - - 2 - - - - 8 - 10 11 -- -- -- 17 18 -- 20 -- 22 -- --
2: 0 1 - - 4 - 6 - 8 9 -- -- -- 15 -- -- 19 20 21 22 23 24
3: - - - - 6 - - 9 -- 11 12 13 -- 15 16 -- 18 -- 20 21 -- --
4: 0 - 2 - 4 - 6 - - 10 -- 13 -- 16 -- -- 20 21 -- 23 --
5: - - - - - 7 - 9 -- -- 13 14 15 16 -- 18 -- 20 21 -- --
6: 0 - 2 3 4 - - 7 8 - -- 12 -- 15 -- -- -- 22 23 --
7: 0 - - - - 5 6 - - - 11 -- 13 14 15 -- 17 18 -- -- 23 --
8: - 1 2 - - 6 - - - -- 12 13 14 -- 16 -- 18 -- -- 22 -- 24
9: 0 - 2 3 - 5 - - - -- -- -- -- 18 19 -- 21 -- -- 24
10: 0 1 - - 4 - - - -- 11 12 -- 14 15 16 17 -- 20 -- 22 -- --
11: - 1 - 3 - - 7 - - 10 -- 12 -- -- 15 16 17 18 19 20 21 22 -- 24
12: 0 - - 3 - - 6 - 8 - 10 11 -- -- 15 16 17 -- 19 -- 21 -- -- 24
13: - - - 3 4 5 - 7 8 - -- -- 14 15 -- -- 19 -- 22 23 24
14: 0 - - - - 5 - 7 8 - 10 -- -- 13 14 -- -- -- 21 -- -- 24
15: 0 - 2 3 - 5 6 7 - - 10 11 12 13 -- -- -- 19 20 21 22 -- --
16: 0 - - 3 4 5 - - 8 - 10 11 12 -- -- -- -- 20 -- -- --
17: - 1 - - - - 7 - - 10 11 12 -- -- -- 18 19 20 21 -- 23 --
18: - 1 - 3 - 5 - 7 8 9 -- 11 -- -- -- 17 -- 19 -- 21 -- 23 --
19: - - 2 - - - - 9 -- 11 12 13 -- 15 -- 17 18 -- -- 22 23 --
20: - 1 2 3 4 5 - - - 10 11 -- -- 15 16 17 -- -- 21 -- 23 --
21: - - 2 3 4 5 - - 9 -- 11 12 -- 14 15 -- 17 18 -- 20 -- 23 --
22: 0 1 2 - - 6 - 8 - 10 11 -- 13 -- 15 -- -- 19 -- -- --
23: - - 2 - 4 - 6 7 - - -- 13 -- -- 17 18 19 20 21 -- -- 24
24: - - 2 - - - - 8 9 -- 11 12 13 14 -- -- -- -- -- -- 23 --

```

Figure 12: Example of "undirected regular" graph

0:	-	-	-	-	-	-	7	-	-	10	11	12	--	--	--	--	--	--	--	--	22	23	24			
1:	-	-	-	-	-	-	7	8	-	10	--	--	--	--	--	--	18	--	--	--	22	23	--			
2:	-	-	-	-	-	6	7	-	-	10	--	--	--	14	--	--	--	--	--	--	--	23	--			
3:	-	-	-	-	-	-	7	-	-	10	--	--	--	--	--	16	--	--	--	20	--	22	23	24		
4:	-	-	-	-	-	-	7	8	-	10	--	--	--	--	--	--	--	--	--	--	--	22	23	24		
5:	-	-	-	-	-	-	7	-	-	10	--	--	--	--	--	--	--	18	--	--	--	22	23	24		
6:	-	-	2	-	-	-	7	-	-	10	--	--	--	--	--	--	--	--	--	--	--	22	23	24		
7:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
8:	-	1	-	-	4	-	-	7	-	-	10	--	--	--	--	--	--	--	--	--	--	--	23	--		
9:	-	-	-	-	-	-	7	-	-	10	--	--	--	--	15	--	--	--	--	--	--	--	22	23	24	
10:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
11:	0	-	-	-	-	-	7	-	-	10	--	--	--	--	--	--	--	--	--	--	--	--	23	--		
12:	0	-	-	-	-	-	7	-	-	10	--	--	--	--	--	--	17	--	--	--	--	--	22	23	--	
13:	-	-	-	-	-	-	7	-	-	10	--	--	--	--	--	--	--	--	--	20	--	--	22	23	24	
14:	-	-	2	-	-	-	7	-	-	10	--	--	--	--	--	--	--	--	--	--	21	--	23	--		
15:	-	-	-	-	-	-	7	-	9	10	--	--	--	--	--	--	--	--	--	--	--	--	22	23	24	
16:	-	-	-	3	-	-	7	-	-	10	--	--	--	--	--	--	--	--	--	--	--	--	--	23	--	
17:	-	-	-	-	-	-	7	-	-	10	--	12	--	--	--	--	--	--	--	--	--	--	--	23	24	
18:	-	1	-	-	-	5	-	7	-	-	10	--	--	--	--	--	--	--	--	--	--	--	--	22	23	24
19:	-	-	-	-	-	-	7	-	-	10	--	--	--	--	--	--	--	--	--	--	21	22	23	24		
20:	-	-	-	3	-	-	7	-	-	10	--	--	13	--	--	--	--	--	--	--	--	--	--	22	23	24
21:	-	-	-	-	-	-	7	-	-	10	--	--	--	14	--	--	--	--	--	--	19	--	--	22	23	24
22:	0	1	-	3	4	5	6	7	-	9	10	--	12	13	--	15	--	--	18	19	20	21	22	23	24	
23:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
24:	0	-	-	3	4	5	6	7	-	9	10	--	--	13	--	15	--	17	18	19	20	21	22	23	24	

Figure 13: Example of "undirected irregular" graph

## 10.1 Algorithms correctness

Algorithms correctness was checked by passing the algorithm graphs with Hamiltonian cycle and testing if algorithm would confirm existence of Hamiltonian cycle. Graphs with 50 vertices were tested.

Tested graphs can be downloaded from:

[http://figshare.com/articles/Correctness\\_Test/1057640](http://figshare.com/articles/Correctness_Test/1057640).

10 000 "directed regular" graphs are located in directory "CT\_50\_T\_T".

10 000 "directed irregular" graphs are located in directory "CT\_50\_T\_F".

10 000 "undirected regular" graphs are located in directory "CT\_50\_F\_T".

10 000 "undirected irregular" graphs are located in directory "CT\_50\_F\_F".

For every tested graph, algorithm confirmed existence of Hamiltonian cycle.

## 10.2 Algorithms efficiency

Following results were given on AMD Athlon II X4 640 3.00 GHz CPU on Windows 7. Algorithm was tested with every one of 4 types of graphs.  $N$  is number of vertices in graph. Values in cells show number of tested graphs:

N	"directed regular"	"directed irregular"	"undirected regular"	"undirected irregular"	required stage time(seconds)
<b>25</b>	1000000	1000000	1000000	1000000	<b>8.75</b>
<b>50</b>	750000	750000	750000	750000	<b>35</b>
<b>75</b>	500000	500000	500000	500000	<b>78.75</b>
<b>100</b>	250000	250000	250000	250000	<b>140</b>

"Directed regular" graphs are located in directory "E\_A\_T\_T".

"Directed irregular" graphs are located in directory "E\_A\_T\_F".

"Undirected regular" graphs are located in directory "E\_A\_F\_T".

"Undirected irregular" graphs are located in directory "E\_A\_F\_F"

where A is number of vertices.

Tested graphs can be downloaded from:

[http://figshare.com/authors/Pawe\\_Kaftan/568545](http://figshare.com/authors/Pawe_Kaftan/568545).

## 11 Algorithms implementation

Algorithms implementation in C#:

<http://findinghamiltoniancycle.codeplex.com/>

or

[http://figshare.com/articles/Implementation\\_Necessary\\_Code/1057722](http://figshare.com/articles/Implementation_Necessary_Code/1057722)